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Solution of the Traveler's Dilemma

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Abstract:

Aim: The aim of the article is to show: 1) that the reasoning of perfectly rational players presented in 1994 by the author of the Traveler's Dilemma Kaushik Basu is incorrect and therefore leads to wrong conclusions, 2) how the reasoning of these players should look like and what solution it leads to.

Design / Research methods: Logical analysis.

Conclusions / findings: Perfectly rational Traveler's Dilemma players should use, according to game theory, so-called retrograde (iterative) induction. This is wrong, as in the set of Traveler's Dilemma games results the principle of transitivity is not met. We believe that perfectly rational players will achieve a better result when they make a random decision from a suitably limited set of decisions. After applying this method of decision making, perfectly rational players will achieve a result similar to those obtained by real players in experiments. Thus, the paradox described in the theory of games disappears, that perfectly rational players achieve worse results than real players

Originality / value of the article: A new way of making decisions in the Traveler's Dilemma game.

Implications of the research: A new way of making decisions in other games similar to the Traveler's Dilemma may allow to find new solutions in these games.

Limitations of the research: The described decision-making method can potentially be used in decision-making situations when the following five conditions are met: 1) the set of possible decisions of each player is greater than 2, 2) the winning matrix is known to both players and both know the purpose of their choices, 3) when it is played once with an unknown opponent, 4) when both players have to make their decision without knowing the opponent's choice, 5) when there is no decision, which is a stable balance point or when it is, but its choice means that the player does not achieve a satisfying result.

Key words: game theory, Traveler's dilemma, perfectly rational player, backward induction.

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1. Introduction

In 1994 Kaushik Basu devised The Traveler's Dilemma (TD) (Kaushik Basu 1994: 391-395). It illustrates a situation when two travelers loose identical exotic souvenirs when returning from the holidays. Feeling guilty the airline wants to cover the loss. However, they do not know how much the souvenirs might have cost and suggest the following solution. Both travelers will, independently of each other, write down the value between 2 and 100 dollars on a piece of paper. If they both put down the same number then both of them will receive it. If the numbers are different, the traveler who wrote a lower number will get the amount he put down increased by a bonus of 2 dollars and the other will receive the amount decreased by a penalty of 2 dollars. If the first one writes 100 and the other 48, the first one will be paid 46 dollars (48-2=46) and the other will get 50 (48 +2= 50).

This paper addresses the problem to the players who:

1. are perfectly rational in striving for the best possible result for themselves,
2. both know that they are playing against with a perfectly rational player,
3. play the game only once with a particular player.

According to current game theory, rationally behaving players under these circumstances should write 2 dollars. This results from the following reasoning. If the other traveler puts down 100 on his piece of paper, it is best for the first one to write down 99 because then he will get 101 dollars. If the other one arrives at the identical conclusion and writes 99 instead 100 then the first players should write 98 as he will receive 100 dollars which is the best result provided the competitor put 99 on his piece of paper. If one player assumes that the other decides in favor of 98, it is best for him to bet on 97. If the other player makes up his mind for 97 then automatically 96 becomes the best for the first player.

The so-called backward induction makes both players write down 2 dollars each. This solution is the Nash equilibrium as it does not pay for the player to change his decision so both of them will stand by their decisions. This is called an equilibrium point.

2. Why cannot backward induction be applied in the Traveler's Dilemma?

Backward induction leads to correct conclusions when in a decision set of the game the rule of transitivity is met. Transitivity guarantees that when we compare decision A with B and we see that decision A gives a worse result than B (A is worse than B) then we compare decisions B and C stating that B is worse than C, then we may be sure that when we compare A and C we will arrive at the conclusion that A is always worse than C.

Backward induction in TD makes use of the transitivity rule. We should see it clearly when we describe the way of reasoning of both rational players. If the first player compares the choices of 100 and 99, regardless of the other player's decision, he will conclude that 99 is better than 100 for him as it always gives the same or better result – see Table 1.

Table 1. Matrix fragment of the first player's winnings

		Decisions of player 2						
		100	99	98	97	96	95	from 2 to 94
Decisions of player 1	100	100	97	96	95	94	93	from 0 to 92
	99	101	99	96	95	94	93	from 0 to 92
	98	100	100	98	95	94	93	from 0 to 92
	97	99	99	99	97	94	93	from 0 to 92
	96	98	98	98	98	96	93	from 0 to 92
	95	97	97	97	97	97	95	from 0 to 92

Source: Authors' own elaboration.
The numbers show the result to be achieved by the first player when the other one makes a specific decision.

If the first player rejects 100 and assumes that the other player will do the same then when comparing 99 and 98 he will conclude that 98 is better than 99 as no matter what his competitor will do, he will have the same or better results betting on 98 rather than 99. If we take 99 and 100 out of the decision set and compare 98 and 97 the we may say that for the first player 97 is the best which makes us reject 98 and so on. By doing so we arrive at the conclusion that the best decisions' set consists only of one element namely 2. This reasoning assumes that if 100 is worse than 99 and 99 is worse than 98 then we can remove 99 and 100 from the set of potentially best decisions so none of the players will get back to a rejected decision in a particular moment. If this reasoning is to be logical in TD, the transitivity rule must be met.

Let us check if this is the way in TD. When we were previously comparing two neighboring decisions we concluded that 100 was always worse than 99 then 99 was worse than 98, 98 was worse than 97[1]. Thus, we should arrive at the conclusion that 100 must always be worse than 97. Looking at Table 1 we notice that this is not the way it is. 100 is worse than 97 provided the other player does not make up his mind for 100. If we assume that 100 is not the best decision and that is why it will not be chosen then indeed 100 is always worse than the decision about 97. Without making such an assumption we cannot say that 100 is worse than 97. Such reasoning is logically flawed.

The above reasoning proves that in the set of TD decisions the transitivity rule is not complied with so backward induction assuming occurrence of the transitivity rule cannot be applied here.

Conclusion 1: Perfectly rational players in TD cannot apply backward induction as this thinking procedure is wrong. It results from the fact that in the results' set the transitivity rule is not met.

We already know how a perfectly rational player should not reason but we do not know how he should think.

3. How do perfectly rational players reason in TD?

As perfectly rational players share knowledge and are also aware that the game is played once, initial simple conclusions may be drawn.

[1] Looking at Table 1 we can see that 97 is better than 98 only in two cases namely when the other player chooses 98 or 97. In remaining 97 cases the decision about 98 is better or the same as 97.

Conclusion 2. There is at least one way of deciding in each game. A random selection is always available. We define it as one in which the probability of choosing each decision from a given set is the same. Thus, we may say that if in TD decisions are made non-randomly, we have at least two ways of making a decision.

The question arises: in what way do perfectly rational players choose a method of arriving at a decision? It does not concern the choice of a decision but the way of establishing which decision is the best for the player. The answer is simple. They compare alternative ways of making a decision and select the one which secures achievement of the best result in a particular game.

The fact that rational players will compare various ways of making a decision with a random selection in TD limits the area of selecting a decision. To understand why, let us first establish what result may be achieved in TD when both players randomly choose a decision out of 2-100 range. If both players randomly choose their decision, the probability of drawing each number from a given set must be the same. Then the effects of such a decision are best shown by the expected value of winning calculated with the same probability of choosing each number from the set of all possible decisions. Thus, we apply the Laplace criterion.

If both players randomly choose from the set of 2-100, the value of the expected winning will go up to 34.50168[2]. The expected value of winning does not guarantee that each player achieves such a result in a particular game but it informs us that if such a player played TD infinitely many times then the average winning in a single game would amount to 34.50168. When making a decision randomly the anticipated value is the best measure allowing us to choose the best solution.

This also means that it does not pay to make a decision non-randomly in favor of 32 or lower because then the maximum winning is 34 which is lower than in case of a random decision. If the two players arrive at such a conclusion, they limit their random choice to the range 33 to 100. Then the anticipated value of the winning must be higher than 34.50168. This in turn once again limits the set of permissible decision taken randomly.

[2] Calculations of the results are shown in appendix 1.

In brief, let us assume that previous reasoning made the players conclude that it did not pay to choose decisions lower than 90. Then their further reasoning will go according to the pattern shown in Table 2.

Table 2. Limiting the selection of decisions made randomly

Decision set	**Anticipated value of the winnings based random selection from a particular set**	**Decisions which may be eliminated from the set**
from 90 to 100	93.181818	90 and 91
from 92 to 100	94.518519	92
from 93 to 100	95.1875	93
from 94 to 100	95.85714	-

Source: Authors' own elaboration.

Let us assume that we have arrived at the conclusion that the selection must be limited to the set (90; … 100). The anticipated value of winnings based on random selection from this set amounts to 93.181818[3]. At the same time, players will figure out that it does not pay to make non-random decisions of 91 and lower as maximum winnings for 91 amounts to 93 which is less than with a random selection of the analyzed set. Thus, the selection area is limited to the set (92; … 100). The anticipated value of the winnings from this set of random decisions amounts to 94.51 which means that 92 may be removed from the set of acceptable decisions. These decisions allow the players to achieve 94 maximum which is less than in case of random selection. When the players limit their selection to the set (93; … 100) then on average they will achieve the result of 95.18 choosing randomly. Similarly, this will eliminate 93. It turns out that this is the last decision which may be eliminated thanks to applied reasoning. This is because the anticipated value of the winnings amounts to 95.85714 when selecting randomly from the set (94; … 100). This result does not let us go on with limiting decisions as this time non-random selection of 94 may result in 96 which is better than the anticipated value of the winnings from the set (94; … 100).

The above reasoning can be called: "Iterated Laplace Rationality by successively reducing the set of actions". We define them as follows:

[3] Calculations of the value are shown in the appendix.

a) Let the set of actions available to each player in a symmetric game be S_0 and set i=0.

b) At stage i calculate the expected reward of each player when both players pick each action from S_i with the same probability, call this expected payoff E_i.

c) S_{i+1} is obtained from S_i by removing all the actions which always give a payoff of less than E_i when the set of actions available is S_i.

d) If S_{i+1} is a strictly smaller set than S_i, then increase i by one and return to step b), otherwise S_i is the set of "Laplacian rational actions by successive reduction".

The above reasoning leads us to the following conclusion.

Conclusion 3. Regardless of the way the decision is made, both players will not go out of set (94; ...; 100). This is a stable point of equilibrium in a sense that if one of the players considers that it is best to limit his decision to set (94; ...; 100) and assumes that his opponent arrives at the same conclusion, it will not still pay to go out of this set and make a decision lower than 94. Thus, we can say that the limitation of the set of decisions to (94: ...: 100) is the Nash equilibrium point.

We already have a narrowed selection field of perfectly rational players and we still do not know in what way they are going to choose the best decision. For the purposes of narrowing the search area let us conduct the following reasoning using the proof by negation.

Let us assume that in TD there is one best non-random method of making a decision and it gives a better result than 95.85714[4]. Then we may be sure that both rational players will reject random selection and apply a non-random method. If both players apply the same method of making a decision, they will have to reach the same decisions. Symmetric pairs from (94; 94)[5] to (100; 100) will become a solution to the game for both players. If any decision of one player meets an identical one of the other one, it does not become his best decision as it is enough a particular player lowers his number by 1 and achieves a better result than

[4] This is an anticipated value of the winnings based on random selection made by both players out of 94 to 100 set.

[5] According to conclusion 3 we know that both players limit their selection by omitting 93 and lower numbers.

previously. Thus we arrived at the contradiction to the assumption we made. This proves that there is no one non-random way of making a decision which guarantees achievement of the best result lower than 95.85714.

The question arises what would happen if there was no best method of making a decision but there would be two or more of them. If there were two non-random ways of taking a decision and if they were equal they would have to lead to achievement of equally good results so both players would be made to arrive at the same decisions. However this is in contradiction to the statement that such a symmetric decision is the best one for a particular player. Thus we put forward the next conclusion.

Conclusion 4. In TD, perfectly rational players who have common knowledge when playing a single game will have to choose randomly from a set consisting of a maximum of 94 to 100 numbers.

Random selection in TD may denote two apparently different actions:

1. a player randomly chooses his decision out of a specific set of possibilities;
2. a player randomly assumes that his opponent will choose a particular decision and will non-randomly adjust his knowledge.

Both actions must be considered random though in the second one there is a random element which decides about the choice. It will be explained in detail in the further part of the paper when we will simulate reasoning of both players.

The fact that both players must randomly select a number from a given set means that we can use the Laplace criterion to evaluate these sets, because the probability of choosing each number from a given set is the same then.

As we already know that both players will have to make a random decision, let us ask a question whether we can narrow down the set (94; … 100) because TD has a rule that the higher the decisions, the higher anticipated value of the winnings. They reach the maximum for the set (99; 100). In random selection the anticipated value of winnings amounts to 99.25. Let us check if perfectly rational players will limit their choice to this set. Such a decision situation is presented in Table 3.

Table 3. Matrix of first player's winnings when both players limit their random choice to 99 and 100.

1st player's decision	2nd player's decision		
	random selection out of (100;99)	100	99
random selection out of (100;99)	99,25	100,5	98
100	98,5	100	97
99	100	101	99

Source: Authors' own elaboration.

Table 3 presents all possible decisions to be taken in the set (99;100). A player may randomly choose a decision out of this set or randomly assume that the opponent will make a specific decision and then non-randomly adjust his best decision. The analysis of the case presented in Table 3 indicates that regardless of the decision his opponent will make, it pays for the first player to non-randomly choose decision for 99. If the first player arrives at such conclusion then his opponent does the same by choosing 99 and his prior choice stops being the best as a he will achieve a better result when he responds with 98.

The assumption that the players will limit their choice to the set (99; 100) must be ruled out which means that both players are certain to extend their decision set to minimum (98; 99; 100).

At this point a doubt may arise why the players do not exclude 100 and 99 which turned out worse than 98 in the above reasoning. The answer is as follows. Those decisions could be excluded permanently if the result set in TD comprised the transitivity rule. However, it is not complied with so we must take into account 100 and 99 as they may turn out better in next situations allowing us to make decisions other than those in (99; 100) range.

Let us examine the situation when both players assume that the opponent selects a decision out of (100;99;98) set. Under these circumstances the player may randomly choose out of (100;99), (99;98), (100;98) or (100;99;98). The anticipated value of winnings when choosing randomly from the first set is the biggest [6] so in Table 4 we will only present this case and we will omit the other two as it does not affect the conclusions made. Apart from random selection we also consider the

[6] See appendix 2.

situation that each player may select each decision from the set. Table 4 presents a matrix of the winnings for the first player.

Table 4. Matrix of the first player's winnings when both players limit their choice to (98; 99;100) set.

1st player's decision	2nd player's decision			
	random selection out of (100;99)	100	99	98
random selection out of (100;99)	99,25	100,5	98	96
100	99	98	97	96
99	100	101	99	96
98	100	100	100	98

Source: Authors' own elaboration.

This time there is no best decision in this decision set for the first and the second player, just like it was in the first case, 98 is the best for the first player except for the situation when the second player bets on 100. The first player is unable to define which decision is best for him without assuming the other player's choice. This means that both players must make a random decision. This in turn denotes that one cannot define whether deciding in favor of 97 – going beyond the analyzed set- is profitable or not. Thus we cannot explicitly define if the players will be willing to extend the set up to (97; 98; 99; 100).

To make sure let us examine the matrix for (97; 98; 99; 100) set. Table 5 shows results of calculation. According to this table it pays the player to non-randomly decide on 97 only if the other one bet on 97 or 98. In remaining three cases 97 is not the best choice for the first player. As long as the first player is unable to define his best decision the other one does not know it either. This way we are certain that one cannot explicitly say whether it pays off to add 97 to (98; 99; 100) set.

Generalizing the aforementioned reasoning we can call the "Iterated Laplace Rationality by successively increasing the set of activities". This procedure can be defined as follows:

a) Let the set of actions corresponding to Pareto optimal payoff vectors in a symmetric game be S_0 and set i=0.

b) At stage *i* calculate the expected reward of each player when both players pick each action from S_i with the same probability, call this expected payoff E_i.

c) S_{i+1} are obtained from S_i by adding any action which gives a payoff of greater than E_i against some action from S_i.

d) If S_{i+1} is a strictly larger set than S_i, then increase *i* by one and return to step b), otherwise S_i is the set of "Laplacian rational actions by successive addition".

Table 5. Matrix of the first player's winnings when both players limit their choice to (97; 98; 99;100) set.

1st player's decision	**2nd player's decision**				
	random selection out of (100;99)	100	99	98	97
random selection out of (100;99)	99.25	100,5	98	96	95
100	98.5	100	97	96	95
99	100	101	99	96	95
98	100	100	100	98	95
97	99	99	99	99	97

Source: Authors' own elaboration.

Let us summarize our considerations about decisions which perfectly rational players will make.

Conclusion 5. We can clearly define that the decision set which the players are going to use to make a decision will randomly at maximum consist of (94;…;100) and minimum (98;…;100).

The reasoning we applied allowed use to significantly narrow the selection field of both players which does not mean that perfectly rational players will not try to narrow it down as the value of expected winnings goes up. However, there must be an additional criterion which will allow them to select from the indicated set of decisions and at the same time it will be a stable point of equilibrium. We did not find such a criterion, therefore the conclusion 5 is the last in the reasoning of perfectly rational players.

To sum up the main conclusions one may state as follows. If TD is once played by perfectly rational players having common knowledge then:

1. They cannot use backward induction in their reasoning as the winnings set does not comply with the transitivity rule.

2. They must make a random decision which in this case means that

a. they will randomly choose from a particular decision set or

b. they will randomly assume a decision to be taken by the other player and select the best decision for themselves.

3. We can clearly say that the choice will be limited to (94;…;100) set at a minimum and (98;…;100) set at a maximum

The decision is often defined as follows: "a conscious, non-random choice of one of the options of future action recognized and recognized as possible" (Koźmiński 2002: 85). Using this definition, we can say that the decision of perfectly rational players in TD will be: random selection of one of the numbers from the set not greater than (94; ...; 100) and not less than (98; ...: 100). In this case, the decision that I randomly choose a number from a given set is a conscious non-random choice of the player. Such a decision is a stable point of equilibrium (Nash equilibrium).

The above definition of the word decision, unfortunately, does not match the TD case, because the player cannot write on the card that randomly selects one of the numbers from the given set. Must enter a specific number. So if we cancel the previous definition of the term decision and specify that in TD the decision is the number entered on the sheet, then the balance ceases to be stable. No matter what number both players put on their cards from a given set, at least one of them always comes to the conclusion that a better result will be achieved when it changes its original decision.

If the player's decision in TD is the number he puts on the card, then there is only one pair of decisions in this game, which we will define as a stable equilibrium point. It is a pair (2; 2). Only then, each player knowing that the competitor entered number 2 on the card will conclude that the best choice for him is to enter on his sheet 2. Both players will then get the result 2. A perfectly rational player, however, has no imposed limitation that he must reach a stable equilibrium point. It has to achieve the best result. If an unstable game solution gives him a better score than 2, he must choose an unstable solution. The conscious decision of both players that each of them will randomly choose the number from the previously indicated set guarantees them that in the worst case they will achieve the result of 92. So, if players are perfectly rational, they must choose this decision.

The reasoning presented is that first the player limits the maximum set of best decisions so that it is a stable equilibrium point (Nash equilibrium) and then randomly chooses one decision from this set, it is most likely only meaningful if the minimum five conditions are met:

1. the set of possible decisions of each player is greater than 2,
2. the winning matrix is known to both players and both know the purpose of their choices,
3. when it is fought once with an unknown opponent,
4. when both players have to make their decision without knowing the opponent's choice,
5. when there is no decision, which is a stable balance point or if it is but its choice means that the player does not achieve the best result.

After fulfilling these conditions, it may turn out that the reasoning described in this article will make perfectly rational players make a decision that will give better results than previously indicated as the best. To illustrate the above theorem with examples deserves a separate article.

The TD solution presented here is not an example of limited rationality, i.e. when we choose the first decision that meets our minimum requirements for winning. The presented reasoning of perfectly rational players seeks to choose the optimal decision, which is, maximizing the player's winnings. In the course of the reasoning presented, none of the players assumed that he would choose a decision that would guarantee him a certain minimum score.

4. Perfectly rational players' decisions and real players' decisions

A lot of experiments have been carried out into TD. Most people chose decisions near 100 (Basu 2007: 75). According to a binding game theory 2 was a rational decision. Thus, the following conclusions were made:

1. Most people behave irrationally in TD.

2. Irrational behavior may produce better results than rational behavior (Basu 2007: 72-74).
3. In TD people are not guided by an egoistic desire to maximize their results but tend to cooperate and prefer solutions providing benefits to both players (Basu 2007: 74).

We have tried in this paper to prove that perfectly rational players will tend to choose decisions near 100 which must be considered rational. Thus, all three theses above become invalid.

Comparing perfectly rational players' decisions with real people's decisions we may formulate a new additional conclusion.

Conclusion 6: Under specific circumstances even people not knowing the method of rational decision are able to make one intuitively but it does not refer to spontaneous decisions but those made after consideration. In other words, in some cases people can sense what the best decision is and cannot logically explain why they consider it the best.

Validity of this thesis is proven by all experiments which were carried out before publishing this paper and in which TD players chose decisions near 100. The described way of reasoning in that the player as a result of the analysis first limits the scope of the best decisions to the smallest set and then cannot unambiguously determine which solution from this set is the best random selection seems to be quite commonly used by real people. This would explain why in real-world experiments players typed numbers close to 100 on cards.

Appendix 1

Let us agree on the amount of the winnings of the player when both players randomly make a decision out of the range 2 to 100. To follow the reasoning in a better way let us begin with a presentation of an abridged matrix of the first player's winnings – see Table 7.

Table 7. Matrix of the first player's winnings when both players randomly make a decision out of 2 to 100 range.

Decisions of player 1	**Decisions of player 2**									
	2	3	4	5	6	7	…	98	99	100
2	2	4	4	4	4	4		4	4	4
3	0	3	5	5	5	5		5	5	5
4	0	1	4	6	6	6		6	6	6
5	0	1	2	5	7	7		7	7	7
6	0	1	2	3	6	8		8	8	8
…										
97	0	1	2	3	4	5		99	99	99
98	0	1	2	3	4	5		98	100	100
99	0	1	2	3	4	5		96	99	101
100	0	1	2	3	4	5		96	97	100

Source: Authors' own elaboration.

Each player may make 99 decisions so the probability of obtaining a particular result at a particular decision amount to 1/99.

In the set of decisions of the first player we may observe occurrence of definite regularities depending on the decision taken by the opponent. Three cases may appear at maximum[7]:

1. if the opponent chooses a decision higher than the decision of the first player,
2. if the opponent chooses exactly the same decision as the first player,
3. if the opponent chooses a lower decision than the first player.

[7] 2 and 100 as extreme decisions are an exception. Only two cases appear then. For decision 2 there will be cases 1 and 2 and for decision 100 there will be 2 and 3.

In the first case player 1 will always attain the same result which equals k+2 where k stands for the decision of the first player. This result will repeat 100-k times. The repeating results are marked with the darkest gray in Table 7. When the first player decides on k=2, the repeating result will amount to k+2=4 and repeats itself 98 times as out of 99 decisions of the other player only one decision will not make the first player achieve the result of 4. For 98, the repeating result of the first player is k+2=100 and it will repeat 100-k which is twice.
In the second case the first player will always attain result k.

In the third situation the first player will achieve a result lower by 2 than the opponent's decision. These results are marked white in Table 7. Looking at the results of the first player we see that they constitute subsequent natural numbers in an arithmetic sequence beginning with zero and we always reach a result lower than decision k by 3. This sequence consists of k-3 elements (excluding 0 which does not affect the total of the sequence). When using a pattern for the total of an arithmetic sequence where:

$$S_n = \frac{(a_1 + a_n)n}{2} \tag{1}$$

$a_1 = 1$, a_n = k-3 and n = k-3[8] and after applying these values to pattern 1 we obtain:

$$S_n = \frac{(1+(k-3))(k-3)}{2}$$

Using the above patterns and previous considerations about the results which the first player will achieve in the first two cases, we may elaborate a pattern for the anticipated value of the first players' winnings when he makes decision k in TD:

$$E(k) = \frac{1}{99}\left(\frac{(1+(k-3))(k-3)}{2} + k + (100-k)(k+2)\right)$$

After transformations we arrive at the following:

$$E(k) = \frac{1}{99}\left(-0{,}5k^2 + 96{,}5k + 203\right). \tag{2}$$

[8] In this case a question may arise with regard to the field of number k. It turns out that we do not have to remove 2 (the lowest decision in TD) as after applying k=2 we receive the sum of 0 and it should be like that. The same sum will appear for k=3.

Applying pattern 2 we may calculate the amount of the anticipated value of the first player's winnings when the two players together randomly choose their decisions out of 2 to 100 set. This value will be marked E(2;…;100). As the player may make 99 decisions then:

$$E(2;\ldots;100)=\frac{1}{99}(E(2)+E(3)+\ldots+E(100))$$

When applying patterns 2 to the above formula and after ordering it we obtain the following:

$$E(2;\ldots;100)=\frac{1}{99}\frac{1}{99}\left(-0{,}5\left(2^2+3^2+\ldots+100^2\right)+96{,}5(2+3+\ldots+100)+99\bullet 203\right) \qquad (3)$$

The pattern for the sum of an arithmetic sequence consisting of subsequent natural numbers from 1 and squared is as follows:

$$S_n^2=1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$$

As we are interested in the sum comprising squares of subsequent natural numbers excluding one, we modify the above pattern by excluding 1 . After applying 100 to n we obtain:

$$S_n^2=1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$$

Using pattern 1 we may calculate the sum of the series of 2 to 100 numbers.

$$S_n=\frac{(a_1+a_n)n}{2}=\frac{(2+100)99}{2}=5049$$

Making use of two last numbers calculated and putting them into pattern 3 we may write down:

$$E(2;\ldots;100)=\frac{1}{9801}(-0{,}5\times 338349+96{,}5\times 5049+99\times 203)$$
$$=34{,}50168350168$$

This way we have calculated the anticipated value of the player's winnings when he together with the opponent randomly chooses his decision out of 2 to 100 set.

Appendix 2

If we are interested in the anticipated value of the winnings based on set 90 to 100 it is easier to present calculations in tables instead of transforming patterns elaborated in appendix 1.

Table 8. Matrix of the first player's winnings when both players randomly make a decision out of 90 to 100 range.

		Decisions of player 2											Anticipated value of the first player's winnings
		90	91	92	93	94	95	96	97	98	99	100	
Decisions of player 1	90	90	92	92	92	92	92	92	92	92	92	92	91.81818182
	91	88	91	93	93	93	93	93	93	93	93	93	92.36363636
	92	88	89	92	94	94	94	94	94	94	94	94	92.81818182
	93	88	89	90	93	95	95	95	95	95	95	95	93.18181818
	94	88	89	90	91	94	96	96	96	96	96	96	93.45454545
	95	88	89	90	91	92	95	97	97	97	97	97	93.63636364
	96	88	89	90	91	92	93	96	98	98	98	98	93.72727273
	97	88	89	90	91	92	93	94	97	99	99	99	93.72727273
	98	88	89	90	91	92	93	94	95	98	100	100	93.63636364
	99	88	89	90	91	92	93	94	95	96	99	101	93.45454545
	100	88	89	90	91	92	93	94	95	96	97	100	93.18181818
Anticipated value of the first player's winnings based on 90 to100 set													**93.18181818**

Source: Authors' own elaboration.

As you can see from Table 8, as a random selection produces an average result of 93.1818, it does not pay for the player to make non-random decision 90 and 91. In both cases the maximum winnings are lower that the anticipated value of winnings so a random selection from (90; …;100) produces a better result than a non-random selection of both decisions. Therefore both players will eliminate 90 and 91 out of the decision set.

Table 9. Matrix of the first player's winnings when both players randomly make a decision out of 92 to 100 range.

		Decisions of player 2									Anticipated value of the first player's winnings
		92	93	94	95	96	97	98	99	100	
Decisions of player 1	92	92	94	94	94	94	94	94	94	94	93.77777778
	93	90	93	95	95	95	95	95	95	95	94.22222222
	94	90	91	94	96	96	96	96	96	96	94.55555556
	95	90	91	92	95	97	97	97	97	97	94.77777778
	96	90	91	92	93	96	98	98	98	98	94.88888889
	97	90	91	92	93	94	97	99	99	99	94.88888889
	98	90	91	92	93	94	95	98	100	100	94.77777778
	99	90	91	92	93	94	95	96	99	101	94.55555556
	100	90	91	92	93	94	95	96	97	100	94.22222222
Anticipated value of the first player's winnings based on 90 to100 set											**94.51851852**

Source: Authors' own elaboration.

We can see that it does not pay the first player to non-randomly decide about 92 as the maximum winnings in this case are lower than the anticipated value with the selection from 92 to 100. That is why both players will eliminate the decision out of this set.

Table 10. Matrix of the first player's winnings when both players randomly make a decision out of 93 to 100 range.

		Decisions of player 2								Anticipated value of the first player's winnings
		93	94	95	96	97	98	99	100	
Decisions of player 1	93	93	95	95	95	95	95	95	95	94.75
	94	91	94	96	96	96	96	96	96	95.125
	95	91	92	95	97	97	97	97	97	95.375
	96	91	92	93	96	98	98	98	98	95.5
	97	91	92	93	94	97	99	99	99	95.5
	98	91	92	93	94	95	98	100	100	95.375
	99	91	92	93	94	95	96	99	101	95.125
	100	91	92	93	94	95	96	97	100	94.75
Anticipated value of the first player's winnings based on 90 to100 set										**95.1875**

Source: Authors' own elaboration.

As with random selection from (93;…;100) set the anticipated value of winnings amounts to 95.1875 it does not pay any of players to non-randomly choose 93 as the maximum winnings comes up to 95. That is why the decision about 93 will be removed from the decisions set.

Table 11. Matrix of the first player's winnings when both players randomly make a decision out of 94 to 100 range.

		Decisions of player 2							Anticipated value of the first player's winnings
		94	95	96	97	98	99	100	
Decisions of player 1	94	94	96	96	96	96	96	96	95.71428571
	95	92	95	97	97	97	97	97	96
	96	92	93	96	98	98	98	98	96.14285714
	97	92	93	94	97	99	99	99	96.14285714
	98	92	93	94	95	98	100	100	96
	99	92	93	94	95	96	99	101	95.71428571
	100	92	93	94	95	96	97	100	95.28571429
Anticipated value of the first player's winnings based on 90 to100 set									**95.85714286**

Source: Authors' own elaboration.

In this case we cannot eliminate 94 as the maximum winnings amounts to 96 which is more than the anticipated value when the decision is randomly made out if (94;…;100) set.

Appendix 3

In order to show that random selection is the best out of (100; 99) set as compared to random section out of (99; 98) and (100; 99; 98) we compare results presented in tables 12, 13 and 14.

Table 12. First player's winnings based on (100; 99) set

Decision of player 1	Decision of player 2	Winnings of player 1	Average winnings for player 1 based on his decision	Average winnings for player 1 based on all his decision
100	100	100	98.5	99.25
	99	97		
99	100	101	100	
	99	99		

Source: Authors' own elaboration.

Table 13. First player's winnings based on (100; 99; 98) set.

Decision of player 1	Decision of player 2	Winnings of player 1	Average winnings for player 1 based on his decision	Average winnings for player 1 based on all his decision
100	100	100	97.66666667	98.55555556
	99	97		
	98	96		
99	100	101	98.66666667	
	99	99		
	98	96		
98	100	100	99.33333333	
	99	100		
	98	98		

Source: Authors' own elaboration.

When comparing the results of the first player we may state that provided the player decides to randomly choose a decision from a particular set, he will obtain the best results when he limits hid choice to (100; 99) set.

Table 14. First player's winnings based on (98; 99) set.

<table>
<tr><th>Decision of player 1</th><th>Decision of player 2</th><th>Winnings of player 1</th><th>Average winnings for player 1 based on his decision</th><th>Average winnings for player 1 based on all his decision</th></tr>
<tr><td rowspan="2">99</td><td>99</td><td>99</td><td rowspan="2">98</td><td rowspan="4">98.375</td></tr>
<tr><td>98</td><td>96</td></tr>
<tr><td rowspan="2">98</td><td>99</td><td>100</td><td rowspan="2">98.75</td></tr>
<tr><td>98</td><td>98</td></tr>
</table>

Source: Authors' own elaboration.

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